

HOSSAM GHANEM

(45) 6.5 The Average Value Of The Function

The Average Value Of f on the interval $[a, b]$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

Example 1

27 May 30,
2006

Find the average value f_{av} of $f(x) = \frac{3}{x^2}$ on interval $[-4, -1]$

Solution

$$f(x) = \frac{3}{x^2} = 3x^{-2}$$

$$\int f(x) dx = \int 3x^{-2} dx = -3x^{-1} + c = \frac{-3}{x}$$

$$\begin{aligned} f_{ave} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{-1-(-4)} \int_{-4}^{-1} 3x^{-2} dx = \frac{1}{-1+4} \left[\frac{-3}{x} \right]_{-4}^{-1} = \frac{1}{3} \cdot (-3) \left[\frac{1}{x} \right]_{-4}^{-1} = - \left(\frac{1}{-1} - \frac{1}{-4} \right) \\ &= 1 - \frac{1}{4} = \frac{3}{4} \end{aligned}$$

Example 2

13 February 19, 1995
16 June 6, 1996

If the average value of $f(x) = x^3 + bx - 2$ on $[0, 2]$ is 4. Find b ,

Solution

$$f(x) = x^3 + bx - 2$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$4 = \frac{1}{2-0} \int_0^2 (x^3 + bx - 2) dx$$

$$\left[\frac{1}{4}x^4 + \frac{1}{2}bx^2 - 2x \right]_0^2 = 8$$

$$\frac{16}{4} + \frac{4}{2}b - 4 = 8$$

$$4 + 2b - 4 = 8$$

$$2b = 8$$

$$b = 4$$

Example 37 June 17,
1993Let $f(x) = \frac{x}{\sqrt{x+1}}$. Find the average value of f on $[0, 3]$ **Solution**

$$f(x) = \frac{x}{\sqrt{x+1}}$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{3-0} \int_0^3 \frac{x}{\sqrt{x+1}} dx = \frac{1}{3} \int_0^3 \frac{x+1-1}{\sqrt{x+1}} dx = \frac{1}{3} \int_0^3 \left(\frac{x+1}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}} \right) dx$$

$$= \frac{1}{3} \int_0^3 \left((x+1)^{\frac{1}{2}} - (x+1)^{-\frac{1}{2}} \right) dx = \frac{1}{3} \left[\frac{2}{3} (x+1)^{\frac{3}{2}} - 2 (x+1)^{\frac{1}{2}} \right]_0^3$$

$$= \frac{1}{3} \left[\frac{2}{3} (4)^{\frac{3}{2}} - 2 (4)^{\frac{1}{2}} - \left(\frac{2}{3} (1)^{\frac{3}{2}} - 2 (1)^{\frac{1}{2}} \right) \right] = \frac{1}{3} \left[\frac{2}{3} \cdot 8 - 2(2) - \left(\frac{2}{3} - 2 \right) \right]$$

$$= \frac{1}{3} \left[\frac{16}{3} - 4 - \frac{2}{3} + 2 \right] = \frac{1}{3} \left[\frac{14}{3} - 2 \right] = \frac{1}{3} \left[\frac{14}{3} - \frac{6}{3} \right] = \frac{1}{3} \cdot \frac{8}{3} = \frac{8}{9}$$

Example 420 January 3
.2001Let $f(x) = x^3 + \sin x$. Find the average value of f on $[-3, 3]$ **Solution**

$$f(x) = x^3 + \sin x$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{3 - (-3)} \int_{-3}^3 (x^3 + \sin x) dx$$

 f is odd function

$$f_{ave} = \frac{1}{6} (0) = 0$$



Example 530 Jan. 12.
2008Find the average value of the function $f(x) = 3x|x - 1|$ on interval $[0, 2]$

Solution

$$f(x) = 3x|x - 1|$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{2-0} \int_0^2 3x|x-1| dx = \frac{1}{2} \int_0^1 -3x(x-1) dx + \frac{1}{2} \int_1^2 3x(x-1) dx$$

$$= \frac{-3}{2} \int_0^1 (x^2 - x) dx + \frac{3}{2} \int_1^2 (x^2 - x) dx = \frac{-3}{2} \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_0^1 + \frac{3}{2} \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_1^2$$

$$= \frac{-3}{2} \left[\frac{1}{3} - \frac{1}{2} - 0 \right] + \frac{3}{2} \left[\frac{8}{3} - 2 - \left(\frac{1}{3} - \frac{1}{2} \right) \right] = \frac{-3}{2} \cdot \frac{2-3}{6} + \frac{3}{2} \left(\frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2} \right)$$

$$= \frac{-3}{2} \cdot \frac{-1}{6} + \frac{3}{2} \left(\frac{16-12-2+3}{6} \right) = \frac{1}{4} + \frac{3}{2} \cdot \frac{5}{6} = \frac{1}{4} + \frac{5}{4} = \frac{6}{4} = \frac{3}{2}$$

$$x-1 \quad \begin{array}{c} \boxed{1} \\ - \quad | \quad + \\ \hline \end{array} \quad 1$$

Example 6

$$\text{Let } f(x) = \begin{cases} 3\sqrt{x} & \text{if } 0 \leq x \leq 4 \\ 2x - 2 & \text{if } 4 < x \leq 6 \end{cases}$$

Find the average value of $f(x)$ on $[0, 6]$

Solution

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{6-0} \int_0^6 f(x) dx = \frac{1}{6} \int_0^4 (3\sqrt{x}) dx + \frac{1}{6} \int_4^6 (2x-2) dx$$

$$= \frac{3}{6} \int_0^4 (x)^{\frac{1}{2}} dx + \frac{2}{6} \int_4^6 (x-1) dx = \frac{1}{2} \cdot \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^4 + \frac{1}{3} \left[\frac{1}{2}x^2 - x \right]_4^6$$

$$= \frac{1}{3} (8 - 0) + \frac{1}{3} \left(\frac{1}{2} \cdot 36 - 6 - \left(\frac{16}{2} - 4 \right) \right) = \frac{8}{3} + (18 - 6 - 8 + 4) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$$

Example 728 January
13. 2007Find the average value f_{av} of $f(x) = 1 + \sqrt{4 - x^2}$ on interval $[-2, 2]$

Solution

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{2 - (-2)} \int_{-2}^2 (1 + \sqrt{4 - x^2}) dx = \frac{1}{4} \int_{-2}^2 dx + \frac{1}{4} \int_{-2}^2 (\sqrt{4 - x^2}) dx$$

$$= \frac{1}{4} \left[x \right]_{-2}^2 + \frac{1}{4} \cdot \frac{1}{2} \pi (2)^2 = \frac{1}{4} (2 - (-2)) + \frac{\pi}{2} = 1 + \frac{\pi}{2}$$

Example 8

22 August

11.2001 A

Let f be a continuous even functions such that $f(x) \geq 0$ for all x in \mathcal{R} .
 If the average value of f on $[0, 3]$, $f_{av} = 5$, find the area of the region
 under the graph of f from $x = -3$ to $x = 3$.

Solution

$$A = \int_a^b f(x) dx = \int_{-3}^3 f(x) dx$$

 f is even

$$A = 2 \int_0^3 f(x) dx \rightarrow (1)$$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$5 = \frac{1}{3-0} \int_0^3 f(x) dx$$

$$\int_0^3 f(x) dx = 15 \rightarrow (2)$$

from (1), (2)

$$A = 2 \cdot 15$$

$$\therefore A = 30$$



Homework

1

If the average value of $f(x) = x^3 + kx^2 - 4$
on $[0, 3]$ is 20, Find k

2

Find the average value of function

$$f(x) = \sqrt{1+x} \quad \text{on interval } [3, 8]$$

8 August 28, 1993

3

Find the average value of the function

$$f(x) = \frac{1}{(3x-1)^{\frac{1}{3}}} \quad \text{over the interval } [0, 3]$$

4

Let $f(x) = \frac{x}{\sqrt{x-1}}$

Find the average value of f on $[2, 5]$

5

Let
$$f(x) = \begin{cases} \frac{6}{\sqrt{x}} & \text{if } 1 \leq x < 9 \\ 2x - 16 & \text{if } 9 \leq x \leq 11 \end{cases}$$

Find the average value of $f(x)$ on $[1, 10]$

6

Find the average value of the function

$$f(x) = 1 + \sqrt{4-x^2} \quad \text{on the interval } [-2, 2]$$

6 January 1993

7

Find the average value of function $f(x) = \sqrt{1-x}$
on interval $[-3, -8]$

Homework

8

Find the average value of the function $f(x) = 2\pi x + \sqrt{1 - x^2}$
on the interval $[-1, 1]$

9

Find the average value of the function

$$f(x) = \frac{1}{(2x + 1)^{\frac{2}{3}}} \quad \text{over the interval } [0, 13]$$

4 May 19, 1992

10

Find the average value f_{av} of $f(x) = \sqrt{9 - x^2}$
on interval $[0, 3]$

11

Let $x > 1$, and suppose that the average value of function f on the interval $[1, x]$ is

$$f_{av} = x^5 \quad . \text{ Find } f(x)$$

23 May 26, 2002

12

Find the average value, f_{av} , of $f(x) = 6x^3 - 5x$ on $[-2, 2]$

24 May 27, 2001

13

Find the average value, f_{av} , of $f(x) = 2x + 6x|x|$ on the interval $[0, 2]$

26 June 7, 2003

10

Let $f(x) = \frac{1}{\pi} \sqrt{1 - x^2}$

Find c such that $f(c)$ is the average value of f on $[0, 1]$

17 January 8, 1997

11

Find the average value f_{av} of

$$f(x) = \sec^2 x \quad \text{over the interval } \left[0, \frac{\pi}{4}\right]$$

38 January 15, 2011

12

41 7 January 2012

[4 Pts.] Find the average value of the function $f(x) = |x - 3|$ on the interval $[1, 4]$